

Vortex Dynamics at the Initial Stage of Resistive Transition in Superconductors with Fractal Cluster Structure

Yuriy I. Kuzmin*

*Ioffe Physical Technical Institute of the Russian Academy of Sciences,
26 Polytechnicheskaya Street, Saint Petersburg 194021 Russia*

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The effect of fractal normal-phase clusters on vortex dynamics in a percolative superconductor is considered. The superconductor contains percolative superconducting cluster carrying a transport current and clusters of a normal phase, acting as pinning centers. A prototype of such a structure is YBCO film, containing clusters of columnar defects, as well as the BSCCO/Ag sheathed tape, which is of practical interest for wire fabrication. Transition of the superconductor into a resistive state corresponds to the percolation transition from a pinned vortex state to a resistive state when the vortices are free to move. The dependencies of the free vortex density on the fractal dimension of the cluster boundary as well as the resistance on the transport current are obtained. It is revealed that a mixed state of the vortex glass type is realized in the superconducting system involved. The current-voltage characteristics of superconductors containing fractal clusters are obtained and their features are studied.

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I. INTRODUCTION

Superconductors containing fractal clusters of a normal phase have specific magnetic and transport properties.^{1,2} The study of their current-voltage (U - I) characteristics permits to get new information on the electromagnetic properties as well as on the nature of a vortex state in such materials. The neighborhood of resistive transition, especially its initial stage where the energy dissipation sets-in, is of special interest. In this region the process of vortex depinning gradually accrues resulting finally in the destruction of superconducting state. High temperature superconductors (HTS's) containing clusters of correlated defects^{3,4} are of special interest in this field. The case of clusters with fractal boundaries provides new possibilities for increasing the critical current value.^{5,6}

Let us consider the superconductor containing inclusions of a normal phase, which are out of contact with one another. We will suppose that the characteristic sizes of these inclusions far exceed both the superconducting coherence length and the penetration depth. A prototype of such a structure is a superconducting wire.

The first generation HTS wires are fabricated following the powder-in-tube technique (PIT). The metal tube is being filled with HTS powder, then the thermal and deformation treatment is being carried out. The resulting product is the wire consisting of one or more superconducting cores sheathed by a normal metal. The sheath endows the wire with the necessary mechanical (flexibility, folding strength) and electrical (the possibility to release an excessive power when the superconductivity will be suddenly lost) properties. At present, the best results are obtained for the silver-sheathed bismuth-based composites, which are of practical interest for energy transport and storage. In view of the PIT peculiarity the first generation HTS wire has a highly inhomogeneous structure.⁷ Superconducting core represents a

dense conglomeration of BSCCO micro-crystallites containing normal-phase inclusions inside. These inclusions primarily consist of a normal metal (silver) as well as the fragments of different chemical composition, grain boundaries, micro-cracks, and the domains of the reduced superconducting order parameter. The volume content of a normal phase in the core is far below the percolation threshold, so there is a percolative superconducting cluster that carries the transport current.

The second generation HTS wires (coated conductors) have multi-layered film structure consisting of the metal substrate (nickel-tungsten alloy), the buffer oxide sub-layer, HTS layer (YBCO), and the protective cladding made from the noble metal (silver). Superconducting layer, which carries the transport current, has the texture preset by the oxide sub-layer. In the superconducting layer there are clusters of columnar defects that can be created during the film growth process as well as by the heavy ion bombardment. Such defects are similar in topology to the vortices; therefore they suppress effectively the flux creep that makes possible to get the critical current up to the depairing value.^{4,8,9}

The paper is organized as follows. The setting of the problem is described as well as vortex dynamics in percolative superconductors in the presence of the fractal clusters is considered in Sec. 2. The current-voltage characteristics of superconductor with fractal cluster structure are obtained and the peculiarities of the resistive transition in such materials are discussed in Sec. 3.

II. MAGNETIC FLUX TRAPPING AND DEPINNING IN PERCOLATIVE SUPERCONDUCTORS

A passage of electric current through a superconductor is linked with the vortex dynamics because the vortices

are subjected to the Lorentz force created by the current. In its turn, the motion of the magnetic flux transferred by vortices induces an electric field that leads to the energy losses. In HTS's the vortex motion is of special importance because of large thermal fluctuations and small pinning energies.¹⁰ Here we will consider the simplified model of one-dimensional line pinning when a vortex filament is trapped on the set of pinning centers.¹¹ Superconductors containing separated normal-phase clusters allow for effective pinning, because the magnetic flux is locked in these clusters, so vortices cannot leave them without crossing the superconducting space. A cluster consists of sets of normal-phase inclusions, united by the common trapped flux and surrounded by the superconducting phase. The magnetic flux remains to be trapped in the normal-phase clusters till the Lorentz force created by the transport current exceeds the pinning force. The flux can be created both by an external source (e.g., during magnetization in the field-cooling regime) and by the transport current (in the self-field regime). As soon as the transport current is turned on, it is added to all the persistent supercurrents, maintaining the constant magnetic flux. When the current is increased, the vortices start to break away from the clusters of pinning force weaker than the Lorentz force created by the transport current. The currents are circulating around the normal-phase clusters through the superconducting loops containing weak links. When the total current through such a link exceeds the critical value, the path becomes resistive and the sub-circuit involved will be shunted by the superconducting paths where weak links are not damaged yet. Magnetic field created by the re-distributed transport current acts via the Lorentz force on the current circulating around. As a result, the magnetic flux trapped therein will be forced out through the resistive weak link, which has become permeable to the vortices.

During this process the vortices will first pass through the weak links, connecting the normal-phase clusters. In this case depinning has percolative character,^{12,13,14} because unpinned vortices move through randomly generated channels created by weak links. Weak links form readily in HTS's due to the intrinsically short coherence length.¹⁵ Depending on the specific weak link configuration each normal-phase cluster has its own depinning current, which contributes to the total statistical distribution of critical currents.

One the other hand, weak links not only allow for the magnetic flux percolation, but they also connect superconducting domains between themselves, maintaining the electrical current percolation. So the composite superconductor with normal-phase clusters represents a percolation system, where both the electric percolation of the supercurrent and the percolation of a magnetic flux may happen. As the transport current is increased, the local currents flowing through ones or other weak links begin to exceed the critical values, therefore some part of them become resistive. Thus, the number of weak links involved in the superconducting cluster is randomly re-

duced so the transition of a superconductor into a resistive state corresponds to breaking of the percolation through a superconducting cluster. The transport current acts as a random generator that changes the relative fractions of conducting components in classical percolative medium,¹⁶ hence the resistive transition can be treated as a current-induced critical phenomenon.

Depinning current of each cluster is related to the cluster size. Larger cluster has more weak links over its boundary, and, consequently, the smaller depinning current. Let us take the area of cluster cross-section as a measure of its size. This value will be called "the cluster area" in the subsequent text. An important feature of normal-phase clusters is that their boundaries can be fractal, i. e. the perimeter of their cross-section and the enclosed area obey the scaling law: $P^{1/D} \propto A^{1/2}$, where D is the fractal dimension of the cluster boundary,¹⁷ which can be fractional. As it was first found in Ref.¹, the normal-phase clusters existing in superconductors can have fractal boundaries, which has significant effect on vortex dynamics. In cited work the fractal dimension was found as a result of geometric probability analysis of the normal-phase clusters contained in HTS films. For this purpose the electron photomicrographs of YBCO films prepared by magnetron sputtering were studied. The normal-phase clusters had the form of columnar inclusions, oriented normally to the film surface. The profiles of cluster sections by the film plane were clearly visible on the photomicrographs, and their geometric probability properties were analyzed. So, instead of self-affine fractals,¹⁷ which the normal phase clusters are, their cross-sections were investigated. The cluster geometric sizes were measured by covering the digitized pictures with a square grid. The found perimeter-area relation exhibited the scaling behavior, which is inherent to fractals, in the range of almost three orders of magnitude in cluster area. The fractal dimension of the cluster boundary was estimated by a slope of the perimeter-area regression line. The perimeter-area scaling is of primary importance here, because porous, random, or highly ramified clusters do not necessarily all are fractals. A fractal cluster has such a property that its characteristic measures (in our case - the perimeter and the enclosed area) have to obey the certain scaling law that includes an exponent named fractal dimension. The scaling perimeter-area behavior means that there is no characteristic length scale over all the range of cluster geometric sizes.

After the start of the vortex motion superconductor switches into a resistive state. In the most practically important case of exponential distribution of the cluster areas, which is realized in the YBCO based film structures,^{1,2} the distribution of critical currents is exponential-hyperbolic:¹⁸

$$f(i) = \frac{2C}{D} i^{-2/D-1} \exp(-C i^{-2/D}) \quad (1)$$

where $i \equiv I/I_c$ is the dimensionless electrical current normalized to the critical current $I_c = \alpha (CA)^{-D/2}$ of the

transition into a resistive state, α is the form factor of the cluster, $C \equiv ((2 + D)/2)^{2/D+1}$ is the constant depending on the fractal dimension, \bar{A} is the average cluster area, and D is the fractal dimension of the cluster boundary.

Let us note that the probability density for the critical current distribution of Eq. (1) is equal to zero at $i = 0$, which implies the absence of any contribution from negative and zero currents. This allows to avoid any artificial assumption about the presence of a vortex liquid, having finite resistance in the absence of transport currents due to presence of free vortices: $r(i \rightarrow 0) \neq 0$. Such an assumption is made, for example, in the case of normal distribution of critical currents.¹⁹

The fractal dimension D sets the perimeter-area scaling relation that is consistent with the generalized Euclid theorem, stating that the ratios of corresponding geometric measures are equal when reduced to the same dimension.¹⁷ The fractal dimension of Euclidean clusters coincides with the topological dimension of a line ($D = 1$), while the dimension of fractal clusters always exceeds their topological dimension ($D > 1$) to reach maximum ($D = 2$) for the clusters of the most fractality. The fractional value of dimension reflects a relationship between characteristic measures (in what follows, the perimeter and the enclosed area) of the object with highly indented boundary.

III. CURRENT-VOLTAGE CHARACTERISTICS OF SUPERCONDUCTORS WITH FRACTAL CLUSTERS OF A NORMAL PHASE

The voltage drop across a superconductor in the resistive state is an integral response of all clusters to the transport current:

$$u = r_f \int_0^i (i - i') f(i') di' \quad (2)$$

where u is the dimensionless voltage and r_f is the dimensionless flux flow resistance. The dimensional voltage U and flux flow resistance R_f are related to the corresponding dimensionless quantities u and r_f by the relationship: $U/R_f = I_c(u/r_f)$.

Using the convolution integral of Eq. (2), we can find the U - I characteristics of a superconductor containing fractal clusters of a normal phase²⁰

$$u = r_f \left(i \exp(-C i^{-2/D}) - C^{D/2} \Gamma\left(1 - \frac{D}{2}, C i^{-2/D}\right) \right) \quad (3)$$

The typical curves are shown in Fig. 1. This figure demonstrates that in the range of the currents $i > 1$

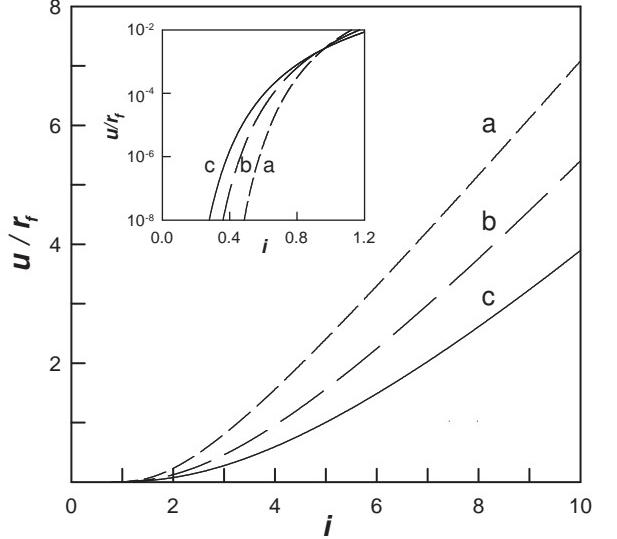


FIG. 1: Current-voltage characteristics of superconductor containing fractal clusters of a normal phase for different values of a fractal dimension: (a) - $D = 1$, (b) - $D = 1.5$, (c) - $D = 2$. The inset shows the initial part of the chart on a semi-logarithmic scale.

the fractality of the clusters reduces the voltage arising from the motion of the magnetic flux transferred by vortices. Meanwhile, the situation is quite different in the neighborhood of the resistive transition below the critical current. When $i < 1$, the higher the fractal dimension of the normal-phase cluster is, the larger is the voltage across a sample and the more stretched is the region of initial dissipation in U - I characteristic. The situation is illustrated in the inset of Fig. 1 where this region is shown on a semi-logarithmic scale.

The significant difference in U - I characteristic behavior below and above the resistive transition is related to the dependence of free vortex density on the fractal dimension for various transport currents. The resistive characteristics provide additional information about the nature of the vortex state at this stage of resistive transition. The standard parameters in this case are dc (static) resistance $r \equiv u/i$, and ac (differential) resistance $r_d \equiv du/di$. The corresponding dimensional quantities R and R_d can be found using the formulas $R = r R_f / r_f$ and $R_d = r_d R_f / r_f$, where R_f is the dimensional flux flow resistance.

For exponential-hyperbolic critical current distribution of Eq. (1) the resistance of a superconductor with fractal normal-phase clusters is given by the formulas:

$$r = r_f \left(\exp(-C i^{-2/D}) - \frac{C^{D/2}}{i} \Gamma\left(1 - \frac{D}{2}, C i^{-2/D}\right) \right) \quad (4)$$

$$r_d = r_f \exp(-C i^{-2/D}) \quad (5)$$

where $\Gamma(\nu, z)$ is the complementary incomplete gamma-function. In the limiting cases of the Euclidean clusters ($D = 1$) and the clusters of the most fractal boundaries ($D = 2$), the above formulas can be simplified:

(a) Euclidean clusters ($D = 1$):

$$r = r_f \left(\exp\left(-\frac{3.375}{i^2}\right) - \frac{\sqrt{3.375\pi}}{i} \operatorname{erfc}\left(\frac{\sqrt{3.375}}{i}\right) \right)$$

$$r_d = r_f \exp\left(-\frac{3.375}{i^2}\right)$$

where $\operatorname{erfc}(z)$ is the complementary error function, and

(b) Clusters of the most fractal boundaries ($D = 2$):

$$r = r_f \left(\exp\left(-\frac{4}{i}\right) + \frac{4}{i} \operatorname{Ei}\left(-\frac{4}{i}\right) \right)$$

$$r_d = r_f \exp\left(-\frac{4}{i}\right)$$

where $\operatorname{Ei}(z)$ is the exponential integral function.

Since the U - I characteristic of Eq. (3) is nonlinear, the dc resistance of Eq. (4) is not constant and depends on the transport current. The more convenient parameter is the differential resistance, a small-signal parameter that gives the slope of the U - I characteristic. Figure 2 shows the graphs of the differential resistance as a function of transport current for superconductor with fractal normal-phase clusters. The curves drawn for the Euclidean clusters ($D = 1$) and for the clusters of the most fractal boundaries ($D = 2$) bound a region containing all the resistive characteristics for an arbitrary fractal dimension. As an example, the dashed curve shows the case of the fractal dimension $D = 1.5$. The dependencies of resistance on the current shown in Fig. 2 are typical of the vortex glass, inasmuch as the curves plotted on a double logarithmic scale are convex and the resistance tends to zero as the transport current decreases, $r_d(i \rightarrow 0) \rightarrow 0$, which is related to the flux creep suppression.^{10,19} A vortex glass represents an ordered system of vortices without any long-range ordering. At the same time, the vortex configuration is stable in time and can be characterized by the order parameter of the glassy state.^{21,22} In the H - T phase diagram, mixed state of the vortex glass type exists in the region below the irreversibility line. The dashed horizontal line at the upper right of Fig. 2 corresponds to a viscous flux flow regime ($r_d = r_f = \text{const}$), which can only be approached asymptotically.

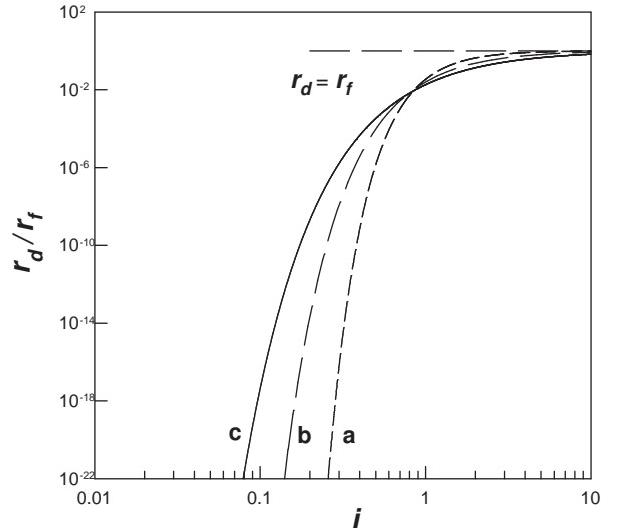


FIG. 2: Dependence of the differential resistance of superconductor with fractal clusters of a normal phase on a transport current for different values of a fractal dimension: (a) - $D = 1$, (b) - $D = 1.5$, (c) - $D = 2$. The dashed horizontal line at the upper right $r_d = r_f$ corresponds to the flux flow regime.

The resistance of superconductor is determined by the density n of free vortices broken away from pinning centers by the transport current i

$$n(i) = \frac{B}{\Phi_0} \int_0^i f(i') di' = \frac{B}{\Phi_0} \exp\left(-C i^{-2/D}\right) \quad (6)$$

where B is the magnetic field, $\Phi_0 \equiv hc/(2e)$ is the magnetic flux quantum, h is the Planck constant, c is the speed of light, and e is the electron charge. The more vortices are free to move, the stronger is the induced electric field, and therefore, the higher is the voltage across a sample at the same transport current. A comparison of expressions of Eqs. (5) and (6) shows that the differential resistance is proportional to the density of free vortices: $r_d = (r_f \Phi_0 / B) n$. Resistance of the superconductor in a resistive state is determined by the motion just of these vortices.

Figure 3 demonstrates dependence of the relative density of free vortices $n(D)/n(D = 1)$ (relatively to the value for clusters with Euclidean boundary) on the fractal dimension for different values of transport currents. The vortices are broken away from pinning centers mostly when $i > 1$, that is to say, above the resistive transition. Here the free vortex density decreases with increasing the fractal dimension. Such a behavior can be explained by the fact that the critical current distribution of Eq. (1) broadens out, moving towards greater magnitudes of current as the fractal dimension increases. It means that more and more clusters of high depinning current, which can trap the vortices best, are being involved in the process. The smaller part of the vortices is free

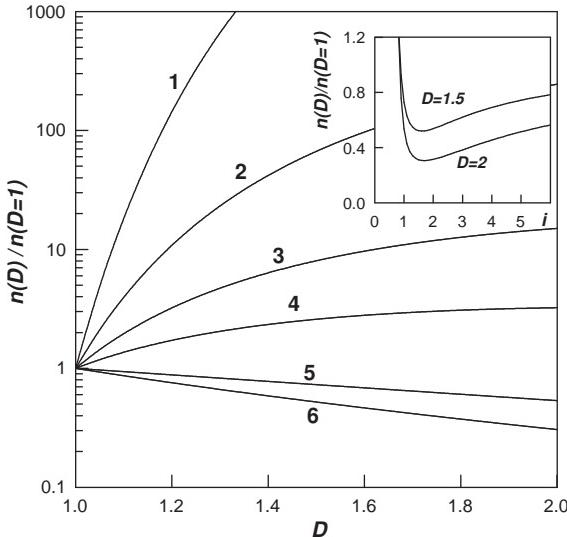


FIG. 3: Dependence of the free vortex density on the fractal dimension for different values of a transport current: (1) - $i = 0.4$, (2) - $i = 0.5$, (3) - $i = 0.6$, (4) - $i = 0.7$, (5) - $i = 1$, (6) - $i = 1.6875$. The inset shows the free vortex density versus current for two different values of a fractal dimension.

to move, the smaller is the induced electric field. An important feature of the fractal superconducting structures is that fractality of cluster boundary enhances pinning⁶ and, hence, a current-carrying capability of the superconductor. This is demonstrated both in Fig. 2, where resistance decreases with increasing fractal dimension above the resistive transition. The relative change in free vortex density depends on the transport current (see inset in Fig. 3) and in the limiting case of the most fractal boundary $D = 2$ reaches a minimum for $i = 1.6875$ (curve 6 in Fig. 3 goes below others). That corresponds to the maximum pinning gain and the minimum level of dissipation. As may be seen in Fig. 1, the voltage across a sample carrying the same transport current decreases with increasing fractal dimension.

In the range of transport currents below the resistive transition ($i < 1$), the situation is different: resistance, as well as the free vortex density, increase for the clusters of greater fractal dimension (see Figs. 2 and 3). Such a behavior is related to the fact that the critical current distribution of Eq. (1) broadens out, covering both high and small currents as the fractal dimension increases. For this reason, the breaking of the vortices away under the action of transport current begins earlier for the clusters of greater fractal dimension. In spite of sharp

increase in relative density of free vortices (Fig. 3), the absolute value of vortex density in the range of currents involved is very small (much smaller than above the resistive transition). So the vortex motion does not lead to the destruction of superconducting state yet, and the resistance remains very low. The low density of vortices at small currents is related to the peculiarity of exponential-hyperbolic distribution of Eq. (1). This function is so “flat” in the vicinity of the coordinate origin that all its derivatives are equal to zero at the point of $i = 0$: $d^k f(0)/di^k = 0$ for any value of k . This mathematical feature has a clear physical meaning: so small a transport current does not significantly affect the trapped magnetic flux because there are scarcely any pinning centers of such small critical currents in the overall statistical distribution, so that nearly all the vortices are still pinned. This interval corresponds to the so-called initial fractal dissipation regime, which was observed in BPSCCO samples with silver inclusions as well as in polycrystalline YBCO and GdBCO samples.²³ As for any hard superconductor some dissipation in a resistive state does not mean the destruction of phase coherence yet. Dissipation always accompanies any motion of vortices that can happen in hard superconductor even at low transport current. Superconducting state collapses only when a growth of dissipation becomes avalanche-like as a result of thermomagnetic instability.

IV. CONCLUSION

The fractal properties of the normal-phase clusters significantly affect the resistive transition. This phenomenon is related to the features of the critical current distribution. The current-voltage and resistance characteristics of superconductor with fractal cluster structure correspond to a mixed state of the vortex glass type. An important result is that the presence of the fractal clusters of a normal phase in a superconductor enhances the pinning. This feature provides the principally new possibility for increasing the current-carrying capability of a superconductor without changing of its chemical composition.

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* Electronic address: yurk@mail.ioffe.ru

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